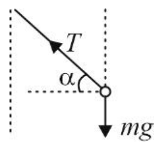
Solutions to JEE MAIN – 7 | JEE 2024

PHYSICS

SECTION-1

1.(A) $T\cos\theta = \frac{mv^2}{r}$



$T\sin\theta = mg$; $\tan\theta = \frac{gr}{v^2}$

$v^2 = \frac{gr}{\tan\theta} = \frac{gr}{\sqrt{3}}$; $v = \left(\frac{gr}{\sqrt{3}}\right)^{1/2}$

2.(B) $F_{\max} = M_{\text{system}} \times a_{\max}$

$= (1+2)\left(\frac{\mu(1)(g)}{1}\right) \quad = 3\times 0.6\times 10 = 18N$

3.(A) A-(II)
Velocity is increasing and positive.
B-(IV)
Velocity is negative and magnitude is decreasing.
C-(III)
Velocity is positive constant initially and then negative constant.
D-(I)
Velocity is a positive constant.

4.(D) $\lambda_m T = \text{constant}$

5.(A) $\Delta Q = \Delta U + \Delta W$

First process adiabatic expansion

$\Delta Q = 0, \quad \Delta W = +50J, \quad \Delta U = -50J$

Second process cooling at constant volume

$\Delta Q = -20J, \quad \Delta W = 0, \quad \Delta U = -20J$

$\Delta U_{total} = (-50) + (-20) = -70$

6.(D) $T = 2\pi\left(\frac{\ell}{g}\right)^{1/2}$

$g_{\text{at height}} < g_{\text{at surface}}$ $\quad \therefore$ A is incorrect.

7.(C) We know that $\quad P_B = P_A + \frac{1}{2}\rho\omega^2 a^2$

$P_D = P_A + \rho g a$

$P_C = P_D + \frac{1}{2}\rho\omega^2 a^2 = P_A + \rho g a + \frac{1}{2}\rho\omega^2 a^2$

Therefore, $\quad P_C > P_A$ for all values of ω and $P_B > P_D$ only if $\omega > \sqrt{\frac{2g}{a}}$

8.(C) Conserving energy between the surface and height R above the surface,

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m\left(\frac{v}{4}\right)^2 - \frac{GMm}{2R} \Rightarrow \quad v = \sqrt{\frac{16GM}{15R}}$$

Let the maximum height above the surface that the object reaches be h

Then, conserving energy between the surface and the maximum height,

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h} \Rightarrow \quad \frac{8GM}{15R} - \frac{GM}{R} = -\frac{GM}{R+h} \Rightarrow \quad h = \frac{8R}{7}$$

9.(B) Let the magnitudes of the forces be F and $30 - F$

Then, $\quad F^2 + (30 - F)^2 = 650$

Solving, we get $F = 25$ N (or 5 N)

Therefore, the forces have magnitudes 25 N and 5 N

So, when the forces are applied at an angle 60° with each other, their resultant is

$$R = \sqrt{25^2 + 5^2 + 2(25)(5)\cos 60^\circ} = 5\sqrt{31}\ \text{N}$$

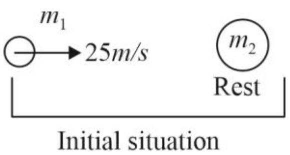
10.(A) $F = -\frac{dU}{dx} = -ve$ of slope of $U - x$ curve

$\therefore$ At P, slope $= -ve \Rightarrow$ force $= +ve$

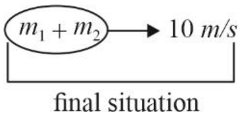
At Q, slope = zero $\Rightarrow$ Force $= 0$

At R, slope $= +ve \Rightarrow$ force $= -ve$

11.(A)



$\Rightarrow$

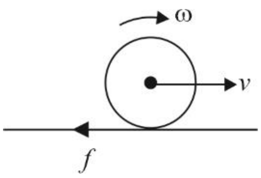


Conserving linear momentum.

$25m_1 + 0 = (m_1 + m_2)10$

$\Rightarrow \quad \frac{m_1 + m_2}{m_1} = 2.5 \quad \Rightarrow \quad \frac{m_2}{m_1} = 2.5 - 1 = 1.5$

12.(C)



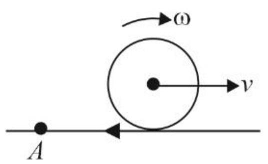
Since in the line of motion 'f' acts as slipping occur

$\therefore$ $\vec{P}$ is not conserved.

Also 'f' changes v and ω both

$\therefore$ Neither translational nor rotational kinetic energy can remain constant.

About A (any point on horizontal surface)



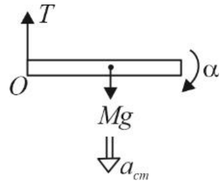
Net $\vec{\tau}_{ext} = 0$

$\because$ τ of N & Mg cancel each other

$\therefore$ $\vec{L}_A =$ conserved

13.(C) Just after the cutting Let $\alpha =$ angular acceleration of Rod & $a_{cm} =$ acceleration of center downwards.

FBD just after the cutting $\Rightarrow$



$\sum F_y = Ma_{cm}$

$\Rightarrow \quad Mg - T = Ma_{cm}$ … (1)

$\sum \tau_0 = I_0 \alpha$

$\Rightarrow \quad Mg\frac{l}{2} = \frac{Ml^2}{3}.\alpha$ … (2)

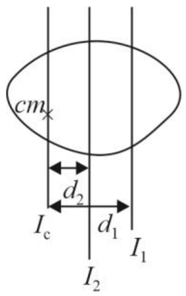
also from constraints, $a_{cm} = \alpha\frac{l}{2}$ … (3)

$\Rightarrow \quad Mg\frac{l}{2} = \frac{Ml^2}{3} \times \frac{2a_{cm}}{l} \Rightarrow a_{cm} = \frac{3g}{4}$

Putting in equation (i)

$\Rightarrow \quad Mg - T = \frac{3mg}{4} \quad \Rightarrow \quad T = \frac{mg}{4}$

14.(D) From parallel axis theorem



$I_1 = I_c + md_1^2$

$I_2 = I_c + md_2^2$

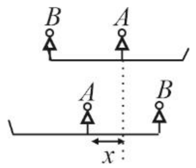
Clearly, $I_c < I_1$

& $I_c < I_2$

Also as $d_1 > d_2$

$I_1 > I_2 \Rightarrow \therefore I_c < I_2 < I_1$

15.(C) Let boat moves to left by $'x'$ as B moves to right end.



$(\Delta x)_{(boat+A)} = (-x)$

$(\Delta x)_B = (10 - x)$

Applying $m_1\Delta x_1 + m_2\Delta x_2 = 0$

$\Rightarrow \quad (100+60)(-x) + 80(10-x) = 0$

$\Rightarrow \quad -240x + 800 = 0$

$\Rightarrow \quad x = \frac{800}{240} = \frac{10}{3}m$

16.(C) Work done in isothermal process,

$W = nRT\log_e\left(\frac{V_2}{V_1}\right) \Rightarrow W = (1)(R)(300)\log_e(8) = (900\log_e 2)R = 900(0.693)(8.31) = 5183\ J$

17.(B) Work = Area under P-V curve $= \frac{1}{2}(3P_0 - P_0)(3V_0 - V_0) + P_0(3V_0 - V_0) = 4P_0V_0$

18.(A) Restoring force, $F = -(kx + 2kx) = -(3k)x$

Therefore, the time period, $T = 2\pi\sqrt{\frac{m}{3k}}$

19.(D) Tension in wire 1, $T_1 = (m_A + m_B)g = 6g$

Tension in wire 2, $T_2 = m_B g = 2g$

We know that $\text{Strain} = \dfrac{\text{Stress}}{\text{Young's modulus}} = \dfrac{T}{AY}$

Therefore, $\dfrac{\text{Strain}_1}{\text{Strain}_2} = \left(\dfrac{T_1}{A_1Y_1}\right)\left(\dfrac{A_2Y_2}{T_2}\right) = \left(\dfrac{T_1}{T_2}\right)\left(\dfrac{A_2}{A_1}\right)\left(\dfrac{Y_2}{Y_1}\right) = \left(\dfrac{6g}{2g}\right)\left(\dfrac{2}{1}\right)^2\left(\dfrac{1}{3}\right) = 4$

20.(A) $I \alpha f^2 a^2 \quad \Rightarrow \quad \dfrac{I_A}{I_B} = 1$

SECTION-2

1.(2) $(V - u) = 10$

$(V + u) = 14$

$2u = 4$

$u = 2$ kmph

2.(200) Distance (D) $= 2R + 2\pi R \times \dfrac{2}{3}$

Time $= \dfrac{D}{v}$

3.(3) Time of flight $= \dfrac{2u\sin\theta}{g} = 2 \times \dfrac{20}{10} \times \dfrac{\sqrt{3}}{2} \qquad = 2\sqrt{3}$ sec

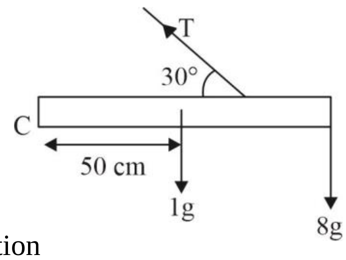
Required time $= \dfrac{T}{2} = \sqrt{3}$ sec

4.(1) $2v = 100 \times 0.02; \qquad v = 1$ m/s

5.(300) $\tau_C = 0$

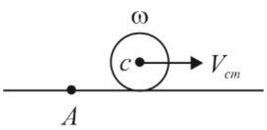
$(T\sin 30°) \times 60 - 2g \times 50 + 8g \times 100$

$T = \dfrac{9000}{30} = 300\,N$



6.(7) For combined translational and rotational motion

We have



$\vec{L}_A = \vec{L}_C + M\left(\vec{R} \times \vec{V}_{cm}\right)$

$L_A = L_C + M.R.V_{cm}$

Now $V_{cm} = R\omega$ and $L_c = (I_{cm})\omega = \dfrac{2}{5}MR^2\omega$

$\Rightarrow \qquad L_A = \dfrac{2}{5}MR^2\omega + MR^2\omega$

$L_A = \dfrac{7}{5}MR^2\omega$

$\therefore \qquad k = 7$

7.(6) For rolling $v = R\omega$

Initial total kinetic energy (Translational + Rotational)

$$= K_i^{tot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{3}\right)MR^2.\frac{v^2}{R^2}$$

$$= \frac{1}{2}Mv^2\left(1+\frac{2}{3}\right) = \frac{5}{6}Mv^2$$

At the top as the sphere just stops $\Rightarrow \quad K_f^{tot} = 0$

Applying energy conservation ($\because$ in pure rolling friction does not do work)

$$\Rightarrow \quad K_i^{tot} + U_i = K_f^{tot} + U_f \quad ; \quad \frac{5}{6}Mv^2 + 0 = 0 + Mgh$$

$$v = \sqrt{\frac{6}{5}gh} \quad \therefore \quad \text{To reach top, } v \text{ should be} \geq \sqrt{\frac{6}{5}gh} \quad \therefore \quad k = 6$$

8.(30) Let the specific heat of the liquids be S_X and S_Y

Then, for the first mixing,

Heat lost by liquid X = Heat gained by liquid Y

$$\Rightarrow \quad 10S_X(80-32) = 20S_Y(32-20) \quad \Rightarrow \quad 2S_X = S_Y$$

Now, let the final temperature after the second mixing be T

So, for the second mixing,

Heat lost by the mixture = Heat gained by liquid Y

$$\Rightarrow \quad (10S_X + 20S_Y)(32-T) = 5S_Y(T-20)$$

Replacing $S_Y = 2S_X$ and solving, we get $T = 30\ ^{o}\text{C}$

9.(43) $\dfrac{dQ}{dt} = \dfrac{KA(T_1 - T_2)}{L} \quad \Rightarrow \quad 1800 = \dfrac{(1)(1.2)(T_1 - 14)}{0.5\times10^{-2}} \quad \Rightarrow \quad T_1 = 21.5\ ^{o}\text{C}$

10.(1) $P = \dfrac{1}{2}\rho\omega^2 A^2 sV$

Since $\dfrac{\lambda_1}{\lambda} = \dfrac{1}{2}, \dfrac{f_1}{f_2} = \dfrac{\omega_1}{\omega_2} = \dfrac{2}{1}$

Since $P_1 = P_2,\ \omega_1 A_1 = \omega_2 A_2,\ \dfrac{A_1}{A_2} = \dfrac{\omega_1}{\omega_2} = \dfrac{1}{2}$

Pressure amplitude $P_0 = B_0 Ak$

$$(P_0)_1/(P_0)_2 = \left(\frac{A_1}{A_2}\right)\left(\frac{k_1}{k_2}\right) = \left(\frac{A_1}{A_2}\right)\left(\frac{\lambda_2}{\lambda_1}\right) \quad = \left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = 1$$

CHEMISTRY

SECTION-1

1.(C) $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$ [One mole of gas at STP occupies 22.4L]

3L 2L

moles $\frac{3}{22.4}$ $\frac{2}{22.4}$

(Limiting reagent)

Finally 0 $\frac{0.5}{22.4}$ $\frac{3}{22.4}$

Mass of H_2O formed $= \frac{3}{22.4} \times 18 = 2.419\,g$

Option (C) is correct.

2.(A) Experiment indicate formation of $FeBr_3$ as single product.

$$M_{FeBr_n} = \frac{W_{FeBr_n} \times 56}{W_{Fe}} = \frac{8 \times 56}{1.5} = 298.66$$

$n = 3$

When 2.00 g of Fe is added, 10.6 of $FeBr_3$ is formed.

When 2.00 g of Fe is added Br_2 is limiting reagent.

When mass of Fe is less than 2.00 g, Fe is limiting reagent.

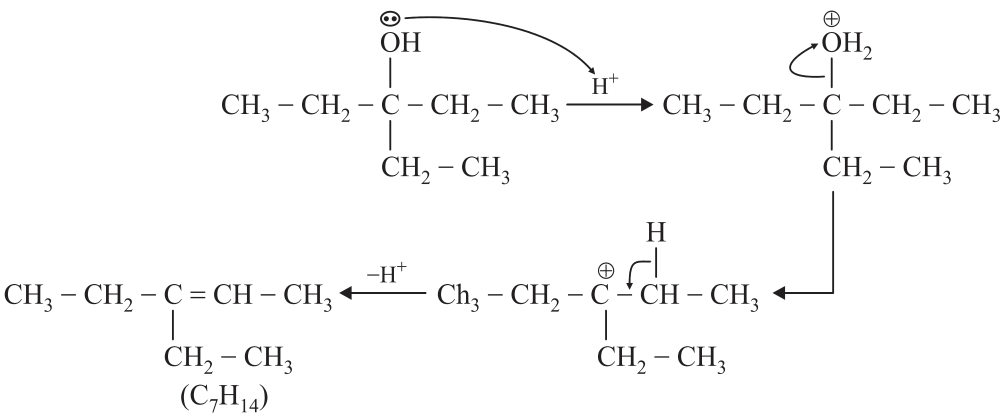
3.(B) Emission of photons of ultraviolet light corresponds to n = 1 final value of the principal quantum number.

4.(A) If shape and orientation of orbital are same then they have same value of ℓ and m_ℓ.

Thus, option A is correct.

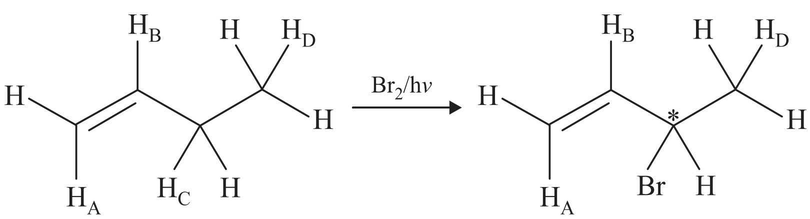
n value may differ corresponding to different number of nodes.

5.(B)



$\therefore$ Only 1 product is obtained from (B).

6.(C)



(Chiral Carbon)

7.(B) $CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)$

$t = t_{eq}$ $\quad 1 - x \quad 1 - 2x \quad x$

$\Rightarrow \quad 2 - 2x = 1.29 \quad \Rightarrow \quad 2(1 - x) = 1.29 \quad \Rightarrow \quad x = 0.355$

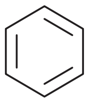
Now, $K_p = \dfrac{p[CH_3OH]}{p[CO] \times p[H_2]^2} = \dfrac{0.355}{0.645 \times (0.29)^2} = 6.54$

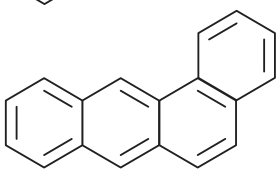
8.(D) $CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(\ell)$; $\quad \Delta_r H^\circ < 0$

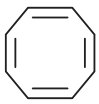
(I) Increasing the temperature, reaction goes to backward direction, because reaction is exothermic. Hence conc. of product is decrease, here yield of CH_3OH decrease.

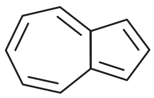
(II) Equilibrium yield does not change by removing some of the $CH_3OH(\ell)$.

9.(A) By gas chromatography highly volatile substances can be easily separated.

10.(A) **(P)**  $\rightarrow$ Benzenoid aromatic

(Q)  $\rightarrow$ Formed on incomplete combustion of organic material

(R)  $\rightarrow$ $8\pi e^-$, does not follow $(4n + 2)\pi e^-$, hence, it is not aromatic

(S)  $\rightarrow$ Non-benzenoid aromatic

11.(B) For precipitate of CaF_2, $[Ca^{2+}] = \dfrac{5.3 \times 10^{-9}}{(0.01)^2} = 5.3 \times 10^{-5}$ M

For precipitate of $Ca_3(PO_4)_2$, $[Ca^{2+}] = \left(\dfrac{1 \times 10^{-25}}{(0.01)^2}\right)^{1/3} = 1 \times 10^{-7}$ M

For precipitate of $CaCO_3$, $[Ca^{2+}] = \dfrac{6.8 \times 10^{-8}}{(0.01)} = 6.8 \times 10^{-6}$ M

Order of $[Ca^{2+}]$ required to start precipitate is $[Ca^{2+}]_{CaF_2} > [Ca^{2+}]_{CaSO_3} > [Ca^{2+}]_{Ca_3(PO_4)_2}$

Hence ion that required least concentration of precipitation reagent will be precipitate first.

12.(C) As balloon deflates work is done by $N_2(\ell)$ on the balloon. Expansion of liquid N_2 results in increase in entropy of the nitrogen.

13.(D) Increase in K_{eq} with temperature indicate that $\Delta_r H^\circ > 0$.

14.(D) For isothermal $\Delta S_{sys} = nR\ell n\dfrac{V_f}{V_i}$.

15.(D) Ionization energy of F is $1681\ kJ\ mol^{-1}$ and of Ar is $1500\ kJ\ mol^{-1}$.

16.(D) Rank of enthalpies:

fusion < vaporization < sublimation

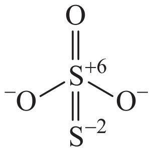
$s \rightleftharpoons \ell$ $\quad$ (fusion)

$(\ell) \rightleftharpoons g$ $\quad$ (vaporization)

$s \rightleftharpoons g$ (sublimation)

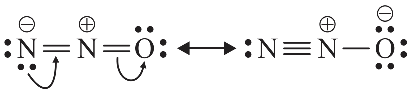
Enthalpy of sublimation is sum of enthalpy of fusion and enthalpy of vaporization.

17.(D)
- Dilution of concentrated acid is exothermic.
- NaOH is readily hydrolysed in water.
- $NaHCO_3$ have low solubility in water.

18.(D) 

19.(B) N_2O is neutral oxide

Resonating structure of N_2O



20.(C) ClF_3 have Trigonal bipyramidal geometry and T-shape. One short equatorial bond and two long axial bonds and $F_a - Cl - F_a$ bond angle of 175°.

SECTION-2

1.(80) $Pb(NO_3)_2 + 2NaBr \rightarrow PbBr_2\downarrow + 2NaNO_3$

3 mmol	3.5 mmol		
3	$\frac{3.5}{2}$ L.R.		
$3-\frac{3.5}{2}$	0	$\frac{3.5}{2}$	3.5

$Pb^{2+} = 1.25$ mmol

$NO_3^- = 1.25 \times 2 + 3.5$

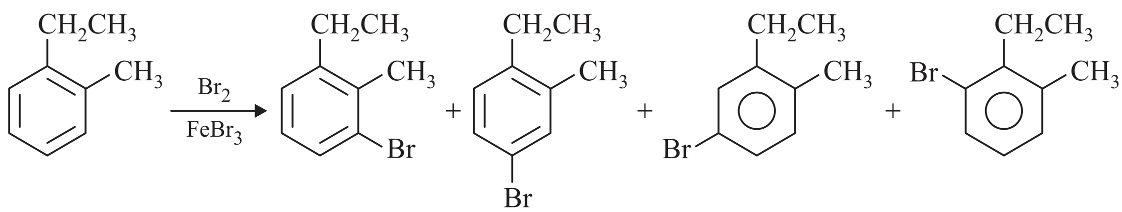
$Na^+ = 3.5$

$Br^- = 0$

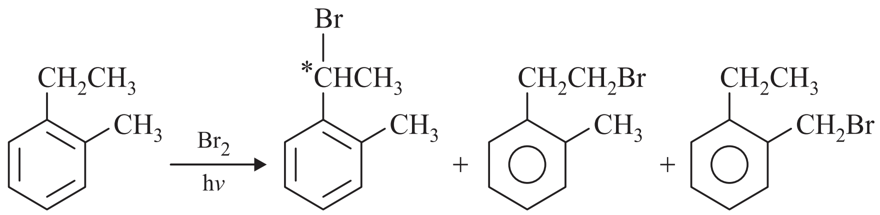
$\therefore$ Br^- is least abundant $\Rightarrow$ Molar mass = 80

2.(6) $n = 3, \ell = 1$

3p-orbital, number of electrons in p-orbital is = 6

3.(8) **(I)** 

N = 4 substitution product are formed

(II) 

M = 4 substitution product are formed

∴ M + N = 4 + 4 = 8

4.(5) $C_3H_6Cl_2$

$$\text{DOU} = 3 - \frac{6}{2} - \frac{2}{2} + 1 = 0$$

(I) Alkylidene halide

$$CH_3-CH_2-\underset{\displaystyle Cl}{\overset{\displaystyle Cl}{\overset{|}{\underset{|}{C}}}}H \qquad CH_3-\underset{\displaystyle Cl}{\overset{\displaystyle Cl}{\overset{|}{\underset{|}{C}}}}-CH_3$$

M = 2 isomers

(II) Asymmetrical molecule

$$CH_3-\underset{\displaystyle Cl}{\underset{|}{\overset{*}{C}}}H-\underset{\displaystyle Cl}{\underset{|}{CH_2}} \qquad \text{(Chiral centre)}$$

N = 2 isomers

(III) Alkylene dihalide

$$CH_3-\underset{\displaystyle Cl}{\underset{|}{C}}H-\underset{\displaystyle Cl}{\underset{|}{CH_2}}$$

P = 1 isomer

∴ Total isomers = 5

5.(13) Let 1 mole KOH be dissolved.

$$\text{Mass of water} = \frac{41.84\times10^3}{4.184\times1} = 10^4\ \text{g}$$

$$V_{solution} = 10\,\text{L}$$

$$[HO^-] = [KOH] = \frac{1}{10} = 0.1\text{M}$$

pOH = 1 ⇒ pH = 14 – pOH = 13

6.(26)

$$AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq) \qquad K_{sp}$$

$$HCN(aq) \rightleftharpoons H^+(aq) + CN^-(aq) \qquad K_a \times 2$$

$$\underline{Ag^+(aq) + 2CN^-(aq) \rightleftharpoons Ag(CN)_2^-(aq) \qquad K_f}$$

$$AgCl(s) + 2HCN(aq) \rightleftharpoons Ag(CN)_2^-(aq) + 2H^+(aq) + Cl^-(aq)$$

AgCl(s)	2HCN(aq)	$Ag(CN)_2^-(aq)$	$2H^+(aq)$	$Cl^-(aq)$
0.01	x	0	0	0
–0.01	–2 × 0.01	+0.01	+2 × 0.01	+0.01
0	(x – 0.02)	0.01	0.02	0.01

$$K_{sp} \times K_a^2 \times K_f = \frac{[Ag(CN)_2^-][H^+]^2[Cl^-]}{[HCN]^2}$$

$$6.78 \times 10^{-7} = \frac{4 \times 10^{-8}}{[HCN]^2} \quad \Rightarrow \quad [HCN] = 0.24 = x - 0.02 \quad \Rightarrow \quad x = 0.26 = 26 \times 10^{-2}$$

7.(333) $2C(s) + 3H_2(g) \longrightarrow \begin{array}{c} \quad H \quad H \\ \quad | \quad\;\; | \\ H - C - C - H \\ \quad | \quad\;\; | \\ \quad H \quad H \end{array}$

$$-84.7 = 2 \times 718.4 + 6 \times 217.9 - (6 \times 416 + x)$$

$$\Rightarrow \quad -84.7 - (2 \times 718.4) - (6 \times 217.9) + (6 \times 416) = -x$$

$$\Rightarrow \quad -x = -333 \quad \Rightarrow \quad x = 333 \text{ kJ mol}^{-1}$$

8.(3) $\overset{+1}{O}Cl^-(aq) + 3I^-(aq) + 2H_3O^+(aq) \rightarrow \overset{-1}{Cl^-}(aq) + I_3^-(aq) + 3H_2O(\ell)$ ($n_f = 2$)

$I_3^-(aq) + 2S_2O_3^{2-}(aq) \rightarrow 3I^-(aq) + S_4O_6^{2-}(aq)$ ($n_f = 1$)

$$= (0.0235 \times 21.23 \text{mL}) = 0.498905 \text{ meq.}$$

meq. of $I_3^- = 0.498905$

meq. of OCl^- = meq. of I_3^-

$$3.33 \times 10^{-x} \times 2 \times 75 = 0.498905$$

$$10^{-x} = 0.000998 \quad \Rightarrow \quad 10^{-x} = 10^{-4} \times 9.9 \simeq 10^{-3} \quad \Rightarrow \quad x = 3$$

9.(6) $Na_3PW_{12}O_{40}$

$$+3 + 5 + 12x - 80 = 0 \quad \Rightarrow \quad 12x - 72 = 0 \quad \Rightarrow \quad x = \frac{72}{12} = 6$$

10.(2) N – F molecule have $7 + 9 = 16e^-$

$\therefore$ 2 unpaired electron in π^* ABMO.

MATHEMATICS

SECTION-1

1.(A) We have $\sec^2\frac{\pi}{7}-\tan^2\frac{\pi}{7}=1$

$\Rightarrow \quad \left(\sec^2\frac{\pi}{7}+\tan^2\frac{\pi}{7}\right)^2-4\sec^2\frac{\pi}{7}\times\tan^2\frac{\pi}{7}=1$

$\Rightarrow \quad \frac{b^2}{a^2}-\frac{4c}{a}=1 \quad \Rightarrow \quad b^2-4ac=a^2 \quad \Rightarrow \quad 4a^2+c^2-4ac=5a^2-b^2+c^2$

2.(B) Let $z=x+iy$

$\Rightarrow \quad x+iy=4\cos^2\theta+4\sin\theta\cos\theta i \quad \Rightarrow \quad x=2(1+\cos 2\theta), y=2\sin 2\theta$

$\Rightarrow \quad x-2=2\cos 2\theta, y=2\sin 2\theta \quad \Rightarrow \quad (x-2)^2+y^2=4$

3.(D) $m=\sum_{r=0}^{\infty}a^r=\frac{1}{1-a}$

$\Rightarrow \quad a=\frac{m-1}{m}$

Similarly, $b=\frac{n-1}{n}$

Equation having roots *a* and *b* is:

$x^2-(a+b)x+ab=0$

or $\quad x^2-\left(\frac{m-1}{m}+\frac{n-1}{n}\right)x+\left(\frac{(m-1)(n-1)}{mn}\right)=0$

or $\quad mnx^2-(2mn-m-n)x+mn-m-n+1=0$

4.(A) Let 1 is associated with *r*.

$r\in\{1,2,3,4,5\}$ then 2 can be associated with *r*, *r* + 1, …., 5.

Let 2 is associated with *j* then 3 can be associated with *j*, *j* + 1, …., 5.

Thus, required number of functions

$=\sum_{r=1}^{5}\left(\sum_{j=r}^{5}(6-j)\right)=\sum_{r=1}^{5}\frac{(6-r)(7-r)}{2}=\frac{1}{2}\left(\sum_{r=1}^{5}\left(42-13r+r^2\right)\right)$

$=\frac{1}{2}\left(42\cdot5-13\cdot\frac{6\cdot5}{2}+\frac{5\cdot6\cdot11}{6}\right)=35$

5.(B) ${}^{2001}C_1\, x^{2000}\frac{1}{2}-{}^{2001}C_2\, x^{1999}\frac{1}{4}+....$

Sum $=\frac{-b}{a}$

6.(A) The highest power of $x=1+2+3+...+12=78$

To get coefficient of x^{70}, we have to omit the factors containing x^8

(1) Product of the constant terms of $(x-1)\left(x^7-7\right)=7$

(2) Product of the constant terms of $(x^2-2)(x^6-6)=12$

(3) Product of the constant terms of $(x^3-3)(x^5-5)=15$

(4) Product of the constant terms of $(x-1)(x^2-2)(x^5-5)=-10$

(5) Product of the constant terms of $(x-1)(x^3-3)(x^4-4)=-12$

Required coefficient = 7 + 12 + 15 – 10 – 12 – 8 = 34 – 30 = 4

7.(D) $32^{33}=2^{165}=2\times16^{41}=2\times(17-1)^{41}=2\times(17k-1)=34k-34+32$

So, the required remainder is 32.

8.(D) $\dfrac{1+\sin 2\alpha}{\cos 2\alpha\cdot\tan\left(\dfrac{\pi}{4}+\alpha\right)}-\dfrac{\sin 2\alpha}{4}\left[\cot\dfrac{\alpha}{2}-\tan\dfrac{\alpha}{2}\right]$

$$=\frac{1+\sin 2\alpha}{\cos 2\alpha\cdot\tan\left(\dfrac{\pi}{4}+\alpha\right)}-\frac{\sin 2\alpha}{4}\left[\frac{\cos^2\dfrac{\alpha}{2}-\sin^s\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}\sin\dfrac{\alpha}{2}}\right]$$

$$=\frac{(\sin\alpha+\cos\alpha)^2}{\cos 2\alpha\left(\dfrac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}\right)}-\cos^2\alpha=\frac{\cos 2\alpha}{\cos 2\alpha}-\cos^2\alpha=\sin^2\alpha$$

9.(A) $S=\{\sin\theta,\sin 2\theta,\sin 3\theta\}$

and $T=\{\cos\theta,\cos 2\theta,\cos 3\theta\}$

Now, $S = T$

This will happen when

$\sin 3\theta=\cos\theta$ ($\sin\theta=\cos 3\theta$ gives the same result)

and $\sin 2\theta=\cos 2\theta$

$\therefore$ $3\theta+\theta=\dfrac{\pi}{2}$ and $2\theta+2\theta=\dfrac{\pi}{2}$ $\therefore$ $4\theta=2n\pi+\dfrac{\pi}{2}$

$\therefore$ $\theta=\dfrac{n\pi}{2}+\dfrac{\pi}{8}, n\in Z$

or

This will happen if

$\sin\theta+\sin 2\theta+\sin 3\theta=\cos\theta+\cos 2\theta+\cos 3\theta$

$\Rightarrow$ $(2\cos\theta+1)(\sin 2\theta-\cos 2\theta)=0$ $\Rightarrow$ $\cos\theta=-\dfrac{1}{2}$ or $\tan 2\theta=1$

$\Rightarrow$ $\theta=2n\pi\pm\dfrac{2\pi}{3}$ (not possible as elements in sets are not equal)

or $2\theta=n\pi+\dfrac{\pi}{4}$

$\Rightarrow$ $\theta=\dfrac{n\pi}{2}+\dfrac{\pi}{8},\ n\in Z$

10.(D) We have $a = 5$, $b = 4$, $c = 3$

I divides AD in the ratio $b + c : a$.

$\therefore$ I divides AD in the ratio 7 : 5 $\quad\quad$ $\therefore$ I is (1, 1)

11.(B) AB subtends right angle at P and Q on variable line.

So, AB is a diameter of circle whose chord is a variable line.

Equation of circle is:

$$x(x-6)+y\times y=0$$

or $\quad x^2+y^2-6x=0 \quad\quad$ ...(i)

Equation of line through (2, 4) is:

$$y-4=m(x-2)$$

or $\quad y=mx+(4-2m) \quad\quad$ ...(ii)

Line (ii) is a chord if $\left|\dfrac{3m+(4-2m)}{\sqrt{1+m^2}}\right|<3$

$\Rightarrow \quad \left|\dfrac{4+m}{\sqrt{1+m^2}}\right|<3 \quad \Rightarrow \quad 16+8m+m^2<9+9m^2 \quad \Rightarrow \quad 8m^2-8m-7>0$

$$m\in\left(-\infty,\frac{2-3\sqrt{2}}{4}\right)\cup\left(\frac{2+3\sqrt{2}}{4},\infty\right)$$

12.(B) $2\left((x-1)^2+(y-2)^2\right)=(x+y+3)^2 \quad \Rightarrow \quad \sqrt{(x-1)^2+(y-2)^2}=\dfrac{|x+y+3|}{\sqrt{2}}$

Therefore, focus is $S(1, 2)$ and directrix is $x + y + 3 = 0$.

Axis of the parabola is $x - y + 1 = 0$.

Solving directrix and axis, we get foot of perpendicular of directrix on axis as $A(-2, -1)$

Hence, vertex is mid-point of AS which is $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$.

13.(B) Clearly locus is ellipse with eccentricity $e=\dfrac{1}{3}$

Here focus is (2, 0) and directrix is $x - 18 = 0$.

$\therefore \quad \dfrac{1}{3}a=2 \quad\quad \therefore \quad a=6 \quad \therefore \quad b^2=a^2\left(1-e^2\right)=36(1-1/9)=32$

$\therefore \quad L.R.=\dfrac{2b^2}{a}=\dfrac{32}{3}$

Alternative method:

Latus Rectum

$= 2e \times$ Distance of focus from directrix $= 2 \times (1/3) \times 16 = 32/3$

14.(D) $S_1P+S_2P=2a$ and $|S_1P-S_2P|=2A$

$S_1P=a+A$ and $S_2P=(a-A)$

Quadratic equation is $x^2-2ax+\left(a^2-A^2\right)=0$

15.(B) $T_r = \dfrac{r^2+3r+3}{r(r+1)(r+2)(r+3)} = \dfrac{r+1}{r(r+2)} - \dfrac{r+2}{(r+1)(r+3)}$

$\therefore \quad T_1 = \dfrac{2}{1\cdot 3} - \dfrac{3}{2\cdot 4}$

$T_2 = \dfrac{3}{2\cdot 4} - \dfrac{4}{3\cdot 5}$

$T_3 = \dfrac{4}{3\cdot 5} - \dfrac{5}{4\cdot 6}$

$\ldots$

$\ldots$

$T_n = \dfrac{n+1}{n(n+2)} - \dfrac{n+2}{(n+1)(n+3)}$

$\therefore \quad S_n = \dfrac{2}{1\cdot 3} - \dfrac{n+2}{(n+1)(n+3)}$

$\therefore \quad S_{10} = \dfrac{2}{1\cdot 3} - \dfrac{12}{11\cdot 13}$

$= \dfrac{250}{429}$

16.(B) Given $\dfrac{a+b}{1-ab}, b, \dfrac{b+c}{1-bc}$ are in A.P.

$\Rightarrow \quad b - \dfrac{a+b}{1-ab} = \dfrac{b+c}{1-bc} - b \quad \Rightarrow \quad \dfrac{-a\left(b^2+1\right)}{1-ab} = \dfrac{c\left(b^2+1\right)}{1-bc}$

$\Rightarrow \quad a + c = 2abc$

Now, given quadratic equation is

$2ac\,x^2 + 2abc\,x + 2abc = 0$

(Substituting $a + c = 2abc$ and then cancelling $2ac$)

$\Rightarrow \quad x^2 + bx + b = 0$

17.(C) $p(x) = \dfrac{1}{24}(x-1)(x-2)(11x^2+31x+24)$

18.(B) $16x^2 + 16y^2 + 48x - 8y - 43 = 0$

or $x^2 + y^2 + 3x - \dfrac{1}{2}y - \dfrac{43}{16} = 0$

Centre of the circle: $\left(-\dfrac{3}{2}, \dfrac{1}{4}\right)$

Radius $= \sqrt{\dfrac{9}{4} + \dfrac{1}{16} + \dfrac{43}{16}} = \sqrt{5}$

Required least distance = distance of line from the centre of the circle – radius of the circle

$= \dfrac{\left|8\left(-\dfrac{3}{2}\right) - 4\left(\dfrac{1}{4}\right) + 73\right|}{\sqrt{64+16}} - \sqrt{5} = \dfrac{60}{4\sqrt{5}} - \sqrt{5} = 2\sqrt{5}$

19. (C) We have

$$\overline{X} = \frac{0^n C_0 + 1^n C_1 + 2^n C_2 + ... + n^n C_n}{{}^nC_0 + {}^n C_1 + {}^n C_2 + ... + {}^n C_n} = \frac{\sum_{r=0}^{n} r^n C_r}{\sum_{r=0}^{n} {}^n C_r}$$

$$= \frac{1}{2^n}\sum_{r=1}^{n} r\frac{n}{r}\ {}^{n-1}C_{r-1} \qquad \left[\because \sum_{r=0}^{n} {}^nC_r = 2^n; {}^nC_r = \frac{n}{r}\ {}^{n-1}C_{r-1}\right]$$

$$= \frac{n}{2^n}\sum_{r=1}^{n} {}^{n-1}C_{r-1} = \frac{n}{2^n} 2^{\,n-1} = \frac{n}{2} \qquad \left[\because \sum_{r=1}^{n} {}^{n-1}C_r = 2^{n-1}\right]$$

and $\frac{1}{N}\sum f_i x_i^2 = \frac{1}{2^n}\sum r^2\ {}^nC_r = \frac{1}{2^n}\sum_{r=0}^{n}\left[r(r-1)+r\right]\ {}^nC_r$

$$= \frac{1}{2^n}\left\{\sum_{r=0}^{n} r(r-1)\ {}^nC_r + \sum_{r=0}^{n} r^n C_r\right\}$$

$$= \frac{1}{2^n}\left\{\sum_{r=2}^{n} r(r-1)\frac{n}{r}\frac{n-1}{r-1}\ {}^{n-2}C_{r-2} + \sum_{r=1}^{n} r\frac{n}{r}\ {}^{n-1}C_{r-1}\right\}$$

$$= \frac{1}{2^n}\left\{n(n-1)2^{n-2} + n2^{n-1}\right\} = \frac{n(n-1)}{4} + \frac{n}{2}$$

$$\therefore \qquad \text{Var}(X) = \frac{1}{N}\sum f_i x_i^2 - \overline{X}^2 = \frac{n(n-1)}{4} + \frac{n}{2} - \frac{n^2}{4} = \frac{n}{4}$$

20.(B) Consider $8^n - 7^n - 1 = (1+7)^n - 7^n - 1$

$$= 1 + 7^n + \left({}^nC_1 7^1 + {}^nC_2 7^2 + ... + {}^nC_{n-1} 7^{n-1}\right) - 7^n - 1$$

$= 7\left({}^nC_1 + 7\ {}^nC_2 + ... + {}^nC_{n-1} 7^{n-2}\right) = a$ multiple of 7 for $n \geq 1$.

For n = 1, $8^n - 7^n - 1 = 0$

$\therefore$ $8^n - 7^n - 1$ is a multiple of 7 for all $n \in$ N.

$\Rightarrow$ X contains elements which are multiple of 7 for $n \in$ N.

Also Y contains elements which are also multiples of 7, but not zero.

SECTION-2

1.(3094) Number of ways in which only one person get his own bag

$= {}^7C_1 \times (\text{Derangement of 6 objects})$

$$= {}^7C_1 \times 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right) = 7 \times 265 = 1855$$

Number of ways in which exactly two person get their own bag

$$= {}^7C_2 \times (\text{Derangement of 5 objects}) = {}^7C_2 \times 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 21 \times 44 = 924$$

Number of ways in which exactly three person get their own bag

$= {}^7C_3 \times (\text{Derangement of 4 objects}) = {}^7C_3 \times 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 35\times 9 = 315$

$\therefore$ Required number of ways = 1855 + 924 + 315 = 3094

2.(154) Coefficient of x in the expansion of $\left(1-2x^3+3x^5\right)\left(1+\frac{1}{x}\right)^8$

$$= \text{Coeff.of } x \text{ in}\left(1+\frac{1}{x}\right)^8 - 2\times \text{Coeff.of } x^{-2} \text{ in}\left(1+\frac{1}{x}\right)^1 + 2\times \text{Coeff.of } x^{-4} \text{ in}\left(1+\frac{1}{x}\right)^8$$

$$= -2\times {}^8C_2 + 3\times {}^8C_4 = -56+210 = 154$$

3.(4) $2\cos 2\theta - 4\cos\theta + 6 = 2\left(2\cos^2\theta - 1\right) - 4\cos\theta + 6$

$$= 4\cos^2\theta - 4\cos\theta + 4 = (2\cos\theta - 1)^2 + 3$$

$-3 \le (2\cos\theta - 1) \le 1$

$\Rightarrow \quad 0 \le (2\cos\theta-1)^2 \le 9 \Rightarrow 3 \le (2\cos\theta-1)^2 + 3 \le 12$

$\Rightarrow \quad \frac{1}{12} \le \frac{1}{(2\cos\theta-1)^2+3} \le \frac{1}{3}$

$\frac{M}{m} = \frac{1/3}{1/12} = 4$

4.(1) $\frac{\sin\alpha}{\sin\beta} = \frac{\cos\gamma}{\cos\delta} \Rightarrow \frac{\sin\alpha - \sin\beta}{\sin\beta} = \frac{\cos\gamma - \cos\delta}{\cos\delta}$ (using divinendo)

$$= \frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{\sin\beta} = \frac{2\sin\left(\frac{\gamma+\delta}{2}\right)\sin\left(\frac{\delta-\gamma}{2}\right)}{\cos\delta}$$

5.(23) For first family the fixed point is (4, –3).

For second family fixed point is $\left(1, -\frac{7}{4}\right)$ and third family has fixed point as $(\lambda-1, 1-2\lambda)$.

Now for three families to have a common member, these three points should be collinear so

$$\frac{1}{2}\begin{vmatrix} 4 & -3 & 1 \\ 1 & -\frac{7}{4} & 1 \\ \lambda-1 & 1-2\lambda & 1 \end{vmatrix} = 0$$

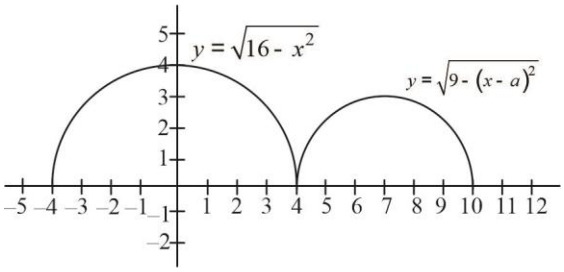
$\Rightarrow \quad 4\left(-\frac{7}{4} - (1-2\lambda)\right) + 3(1-\lambda+1) + \left((1-2\lambda) - \left(-\frac{7}{4}\right)(\lambda-1)\right) = 0 \qquad \Rightarrow \qquad \lambda = \frac{23}{19}$

6.(7) We have

$y = \sqrt{9-a^2+2ax-x^2}$ and $y = \sqrt{16-x^2}$

$\Rightarrow \quad y^2 + (x-a)^2 = 9 \qquad \Rightarrow \qquad y^2 + x^2 = 16$

So the given equations represent semicircles.

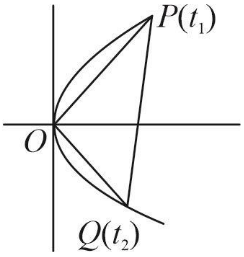


If $a > 7$, no solution exists $\Rightarrow \quad a \le 7$

Hence, max. value of a is 7.

7.(256)

$$PQ = 2\left(t + \frac{1}{t}\right)^2 = 32$$



$$t + \frac{1}{t} = 4, -4$$

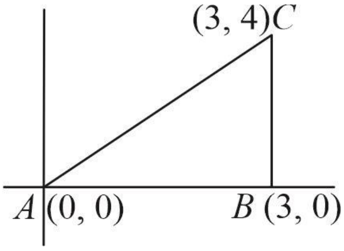
Area of ΔOPQ is = $\frac{1}{2}\begin{vmatrix} 2t^2 & 4t & 1 \\ \frac{2}{t^2} & -\frac{4}{t} & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$$\left|\frac{1}{2}\left(-\frac{8}{t} - 8t\right)\right| = 16$$

8.(6) $AC = 5 = 2ae$

$\Rightarrow \quad 2ae = 5$

Also $AB + BC = 2a$



$\Rightarrow \quad 2a = 7 \qquad \Rightarrow \quad a = \frac{7}{2}$

$\therefore \quad e = \frac{5}{7} \quad \Rightarrow \quad 1 - \frac{b^2}{a^2} = \frac{25}{49} \quad \Rightarrow \quad b = \sqrt{6}$

$$\text{Area} = \pi ab = \pi \times \frac{7}{2} \times \sqrt{6}$$

$\therefore \quad P = 6$

9.(3) $\sin 30° = 3\sin 10° - 4\sin^3 10°$

$\Rightarrow \quad \frac{1}{2} = 3\sin 10° - 4\sin^3 10° \quad \Rightarrow \quad 8\sin^3 10° + b\sin^2 10° - 6\sin 10° + 1 = 0 \quad \ldots(1)$

Given that $f(\sin 10°) = 0$

$\Rightarrow \quad a\sin^3 10° + b\sin^2 10° + c\sin 10° + d = 0 \quad \ldots(2)$

Comparing (1) and (2), we get

$a = 8, b = 0, c = -6, d = 1$

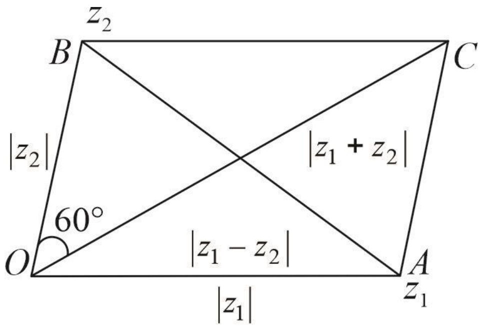
$\Rightarrow \quad f(1) = a + b + c + d = 3$

10.(133)

Using cosine rule

$|z_1 + z_2| = \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos 120°}$

$= \sqrt{4 + 9 - 2\cdot 3} = \sqrt{19}$



and $|z_1 - z_2| = \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_1|\cos 60°}$

$= \sqrt{4 + 9 - 2\cdot 3} = \sqrt{7}$

$\therefore \quad \left|\frac{z_1 + z_2}{z_1 - z_2}\right| = \sqrt{\frac{19}{7}} = \frac{\sqrt{133}}{7} \Rightarrow N = 133$